WEEK 9 : CS275 RECITATION EXERCISES

For each question, read **each word** with the greatest care and **without hurrying**. If you have doubts about what is asked, **go back** to the wording of the question until the meaning of the question is clear. Then try to find an answer.

Exercise 1. P. 319, n. 4 in [1], Section 4.2. A bowl contains ten red balls and ten blue balls. A woman selects balls at random without looking at them.

a) How many balls must she select to be sure of having at least three balls of the same color?
b) How many balls must she select to be sure of having at least three blue balls?

Exercise 2. P. 319, n.6 in [1], Section 4.2.

a) Let d be a positive integer. Show that among any group of $d+1$ (not necessarily consecutive) integers, there are two with exactly the same remainder when they are divided by d.

Exercise 3. P. 319, n. 12 in [1], Section 4.2. How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that $a_1 \bmod 5 = a_2 \bmod 5$ and $b_1 \bmod 5 = b_2 \bmod 5$?[1]

Exercise 4. P. 319, n. 14 in [1], Section 4.2.

a) Show that if seven [distinct] integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11.
b) Is the conclusion of part **a)** true if six integers are selected rather than seven?

Exercise 5. P. 320, n. 40 in [1], Section 4.2. There are 51 houses on a street. Each house has an address between 1000 and 1099, inclusive. Show that at least two houses have addresses that are consecutive integers.

Exercise 6. P. 707, n. 2 in [1], Section 10.1. Find the values, if any, of the Boolean variable x that satisfies these equations.

a) $x \cdot 1 = 0$
b) $x + x = 0$
c) $x \cdot 1 = x$
d) $x \cdot \bar{x} = 1$

Exercise 7. P. 708, n. 7 in [1], Section 10.1. What values of the Boolean variables x and y satisfy $xy = x + y$?

[1] The modulo of a by b, also pronounced "a modulo b" is defined by:

$$a \bmod b = a - b \left\lfloor \frac{a}{b} \right\rfloor \in [0, b) .$$

Exercise 8. P. 708, n. 26 in [1], Section 10.1. Find the duals of these Boolean expressions.

a) $x + y$
b) $\bar{x}\bar{y}$
c) $xyz + \bar{x}\bar{y}\bar{z}$
d) $x\bar{z} + x \cdot 0 + \bar{x} \cdot 1$

Exercise 9. For each of the Boolean functions below,

a) rewrite it in a simpler way, if this is possible;
b) find its sum-of-products expansion, and
c) find its product of sums expansion.

a) $F(x, y) = x + y\bar{x}$
b) $F(x, y, z) = \bar{x}y + \bar{y}z + \bar{z}x$
c) $F(x, y, z) = \bar{x}(yz + \bar{y}\bar{z}) + x(y\bar{z} + \bar{y}z)$
d) $F(x, y, z) = \bar{x}(y + z) + \bar{y}(x + z) + \bar{z}(x + y)$

References

[1] K. H. Rosen. *Discrete Mathematics and Its Applications*. Mc Graw Hill, 5 edition, 2003.