DISCRETE MATH - EXERCISES FOR 2005/09/13

Read **each word** of the question with greatest care and **without precipitation**. If you have some doubts about what is asked, **go back** to the words of the question until the meaning of the question is clear. Then proceed to searching an answer.

Exercise 1.

a) Write in English the following statement about natural numbers:

$$\forall m \, \exists n, \; m - 8n < 7 \, \wedge \, m - 8n \geq 0. \tag{1.1}$$

b) Prove or disprove this statement.

c) Write (in mathematical notation) the negation of proposition (1.1).

Exercise 2.

a) Write in English the following statement about natural numbers:

$$\forall m \, \forall n \; (\exists p, \; n \leq p \, \wedge \, p | m) \Longrightarrow n \leq m. \tag{2.1}$$

b) Prove or disprove this statement.

c) Write (in mathematical notation) the negation of proposition (2.1).

Exercise 3. Let $P(x)$ denote a propositional function on the set of natural numbers. Write in mathematical notation, without using the notation $\exists!$, the proposition:

There exists a unique x that verifies $P(x)$

Exercise 4. Let $\mathbb{R}$ be the set of real numbers. Write in mathematical notation (possibly using $\exists!$) the sentence:

There exists a unique real number x that verifies $ax + b = c$.

Determine whether this statement is true for all strictly positive natural numbers a, b and c. This exercise is somewhat like Ex. 54 p. 76 in [1].

Exercise 5. We have a computer with infinite memory, that runs the following code quicker than you can read it.

```
infinitely long int i;
float [∞] x;
x[0] = 8.0;
for (i=0; i<=∞; i++)
     x[i+1] = 0.5*x[i] + rand()/(RAND_MAX+0.0);

// Note: rand() returns a value between 0 and RAND_MAX.
```

At this point, let´s see what can be said about the contents of the array `x` .

a) Write (possibly useful) upper and lower bounds on `x[1]`, `x[2]`, `x[3]` and `x[4]`.

b) Let $Q(n)$ be the proposition $\texttt{x[}n\texttt{]} \geq 0$. Can you prove (e.g. by induction) that it is true for all n?

c) What about $\texttt{x[}n\texttt{]} > 0$? (treacherous question)

d) Let $P(n)$ be the proposition $\texttt{x[}n\texttt{]} < 3$. Can you show that it is true starting at a certain point? (That is, there exists an N such that $P(n)$ holds for all n greater than N; that is, $n \geq N \Longrightarrow P(n)$).

e) What about the proposition $\texttt{x[}n\texttt{]} \leq 2$? (treacherous question)

Exercise 6. Arithmetic sequence: Write the sum $0+4+8+\ldots+4n$ using the summation symbol $\sum$. What simple expression is this sum equal to?

Exercise 7. Arithmetic sequence: Write the sum $a+(a+3)+(a+6)+\ldots+(a+3n)$ using the summation symbol $\sum$. What simple expression is this sum equal to?

Exercise 8. Geometric sequence: Write the sum $1+4+8+...+4^n$ using the summation symbol $\sum$. What simple expression is this sum equal to?

Exercise 9. Geometric sequence: Write the sum $b+b4+b8+...+b4^n$ using the summation symbol $\sum$. What simple expression is this sum equal to?

References

[1] K. H. Rosen. *Discrete Mathematics and Its Applications.* Mc Graw Hill, 5 edition, 2003.