GRADED HOMEWORK

TO GIVE BACK TUESDAY SEP. 14TH AT BEGINNING OF CLASS.

Exercise 1. Let $A = \{0, 1, 2\}$. Write each of the following statements without using quantifiers, instead using just the $\wedge$, $\vee$ and $\neg$ operators.

a) $\exists x \in A,\ P(x)$
b) $\forall x \in A,\ P(x)$
c) $\exists x \in A,\ \neg P(x)$
d) $\forall x \in A,\ \neg P(x)$
e) $\neg \exists x \in A,\ P(x)$
f) $\neg \forall x \in A,\ P(x)$

Exercise 2. Write the definition of an odd number using mathematical notation.

Exercise 3. Show that the product of two odd numbers is odd (Rosen [1], p. 75, exercise 24).

Exercise 4. Define the set of prime numbers using the "set builder notation," without using the word "prime".

Exercise 5.

a)
  (a) Write in English the following statement about natural numbers:
$$\exists m\, \exists n\, \exists p,\ m^2 + n^2 = p^2.$$
  (b) Prove or disprove this statement.

b)
  (a) Write in English the following statement about natural numbers:
$$\forall m\, \forall n\, \forall p\ (m \mid n\ \wedge\ m \mid p) \Longrightarrow (\forall q\, \forall r,\ m \mid (qn + rp)) .$$
  you may keep the mathematical notation $qn + rp$.
  (b) Prove or disprove this statement.

Exercise 6. Let A and B be two non-empty sets. Prove or disprove that
$$A \times B = B \times A$$
if and only if
$$A = B.$$

Exercise 7. Write the negation of the following proposition without using the existential quantifier $\exists$.
$$\exists m\, \exists n\ (m > 1\ \wedge n > 1\ \wedge\ m^n - n^m = 1)$$

Exercise 8. Let variables range over $\mathbb{Z}$ and let $Q(x,y)$ be the propositional function $x+y=x-y$. Prove or disprove the following propositions (Rosen [1], p. 54, exercise 26):

a) $Q(1,1)$
b) $Q(2,0)$
c) $\forall y\, Q(1,y)$
d) $\exists x\, Q(x,2)$
e) $\exists x\, \exists y\, Q(x,y)$
f) $\forall x\, \exists y\, Q(x,y)$
g) $\exists y\, \forall x\, Q(x,y)$
h) $\forall y\, \exists x\, Q(x,y)$
i) $\forall x\, \forall y\, Q(x,y)$

Exercise 9. Show that, in propositional calculus:

a) $(P \implies Q) \wedge (P \implies R)$ and $P \implies (Q \wedge R)$ are logically equivalent.
b) $\neg P \implies (Q \implies R)$ and $Q \implies (P \vee R)$ are logically equivalent
c) $P \iff Q$ and $\neg P \iff \neg Q$ are logically equivalent.
d) $((P \vee Q) \wedge (\neg P \vee R)) \implies (Q \vee R)$ is a tautology.

(this is Rosen [1], p. 27, exercises 20, 24, 26, 28). You may do so using truth tables or logical equivalence rules.

References

[1] K. H. Rosen. *Discrete Mathematics and Its Applications.* Mc Graw Hill, 5 edition, 2003.