Pronunciation guide for mathematical notation

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$x = y$ reads "x is **equal** to y." Examples: $1 = 1$;$\{3, 5\} = \{3, 5, 3\}$.

$x \neq y$ reads "x is not equal to y" or "x is **different** from y." Examples: $1 \neq 0$;$\{5\} \neq \{3, 5\}$.

$x \in y$ reads "x **belongs** to y" or "x is an **element** of y" or "x is contained in y" or "y **contains** x." Example: $3 \in \{3, 5\}$.

$x \notin y$ reads "x does not belong to y" or "x is not an element of y." Example: $3 \notin \{4, 5\}$.

$x \subseteq y$ reads "x is a **subset** of y" or "x is **included** in y" or "y is a **superset** of x" or "y **includes** x." Example: $\{3, 4\} \subseteq \{3, 6, 4\}$.

$x \subset y$ reads "x is a **proper subset** of y" or "x is a **strict subset** of y" or "x is **strictly included** in y" or "y is a **strict superset** of x." Example: $\{3, 4\} \subset \{3, 6, 4\}$.

$x \cup y$ reads "the **union** of x and y."

$x \cap y$ reads "the **intersection** of x and y."

$x \setminus y$, $\complement_x y$ and $\bar{y}^x$ all read "the **complement** of y in x."

$\forall x$ reads "**for all** x" or "**any** x" or "**given any** x" (this statement does not mean anything alone). The symbol $\forall$ is called the "**universal quantifier**." Examples:

$\forall x,\ x = x$ reads "all object is equal to itself" or
"any object x is equal to itself" or
"for all object x, the statement $x = x$ is true."

$\forall x,\ x \subseteq x$ reads "any object is a subset of itself" or
"all set is a subset of itself"[1] or
"for all set x, the statement $x \subseteq x$ is true."

$\exists x$ reads "**there exists a** x" or "**there is a** x" or "**for some** x" (this statement does not mean anything alone). The symbol $\exists$ is called the "**existential quantifier**." Examples:

$\exists x,\ x \in \{1, 2\}$ reads "there exists an object that belongs to the set $\{1, 2\}$" or
"for some object x, the statement $x \in \{1, 2\}$ is true."

$\exists x,\ x > 3$ reads "there exists an object that is greater than 3" or
"for some x, the statement $x > 3$ is true."

[1]Recall that all mathematical objects are sets.

$\Longrightarrow$ reads "**implies**" or (if preceded by "**if**") "**then**." Examples:

$x = 1 \Longrightarrow x < 2$ reads "if x is equal to 1, then x is smaller than 2" or
"the fact that $x = 1$ implies that $x < 2$" or
"the fact that $x < 2$ is a consequence of the fact that $x = 1$."

$x \in \{1, 2\} \Longrightarrow x < 3$ reads "if x belongs to the set $\{1, 2\}$, then x is smaller than 3" or
"if x belongs to $\{1, 2\}$, then x is smaller than 3."

Alternative notations: $\rightarrow$, $\therefore$.

$\Longleftrightarrow$ reads "**is equivalent to**" or "**if and only if**." Example:

$x \in \{2\} \iff x = 2$ reads "x belongs to the set$\{2\}$ if and only if x is equal to 3" or
"the fact that $x \in \{2\}$ is equivalent to saying that x is equal to 2" or
"saying that x belongs to the set $\{2\}$ is equivalent to saying that x is equal to 2."

Alternative notations: $\equiv$, $\longleftrightarrow$, iff.

$\wedge$ reads "**and**." Example: $x > 1 \wedge x < 4$ reads "x is greater than 1 and x is smaller than 4" or "x is **both** greater than 1 and less than 4." Also, $P \wedge Q$ reads "the **conjunction** of P and Q." Alternative notation: write "and" in full letters. Equivalent `C` operator: `&&`.

$\vee$ reads "**or**." Example: $x < 5 \vee x > 9$ reads "x is smaller than 5 or x is greater than 9" or "x is **either** less than 5 or greater than 9." Also, $P \vee Q$ reads "the **disjunction** of P and Q." Alternative notation: write "or" in full letters. Equivalent `C` operator: `||`.

$\{4\}$ reads "the set containing 4 (and nothing else)" or "the **singleton** having 4 as only element" or "the set containing just 4" or "the set having 4 as only element."

$\{1, 3, 5\}$ reads "the set containing 1, 3 and 5 (and nothing else)."

$\mathbb{N}$ reads "the set of **natural numbers**" or "the set of integers that are positive or zero." $\mathbb{N} = \{0, 1, 2, ...\}$. Examples: $x \in \mathbb{N}$ reads "x belongs to the set of natural numbers" or "x is a natural number"; $3 \in \mathbb{N}$ reads "3 is a natural number"; $\{3\} \notin \mathbb{N}$ reads "the set containing just 3 is not a natural number." $\{3\} \subseteq \mathbb{N}$ reads "the set containing just 3 is a subset of the set of natural numbers."

$\mathbb{Z}$ reads "the set of **integers**". $\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$.

$\{x \in \mathbb{N} \mid x < 3,\ x \neq 1\}$ reads "**the set of** natural numbers **that are** both smaller than 3 and different from 1" or "**the set of** natural numbers x **such that** $x < 3$ and $x \neq 1$." This is: $\{0, 2\}$.

$\{x \in \mathbb{N} \mid x = 1 \textbf{ or } x = 3\}$ reads "the set of natural numbers that are either equal to 1 or equal to 3." Alternative notation: $\{1, 3\}$.

$\emptyset$ reads "the **empty set**." Examples: $\forall x,\ x \notin \emptyset$ reads "for all x, x does not belong to the empty set" or "no object is in the empty set" or "the empty set contains no object;" $\forall x,\ \emptyset \subseteq x$ reads "the empty set is a subset of all sets" or "all sets are supersets of the empty set." Alternative notation $\{\}$.

All the examples given here that do not contain a variable x are true, and all examples that start by $\forall$ or $\exists$ are also true.