CS275 GRADED HOMEWORK 7

GIVE BACK ON TUESDAY NOVEMBER 9TH 2004 AT BEGINNING OF CLASS

For each question, read **each word** with the greatest care and **without hurrying**. If you have doubts about what is asked, **go back** to the wording of the question until the meaning of the question is clear. Then try to find an answer. If you get stuck, don't hesitate to **contact** your T.A. or me.

Please write your section number on your homework as well as a rough estimate of the time you spent solving it.

1. Reminder: Solving recurrence relations

In order to solve the linear homogeneous recurrence relation of degree two

$$x_n = c_1 x_{n-1} + c_2 x_{n-2}, \tag{1.1}$$

you first find the solutions r_1 and r_2 of the characteristic polynomial,

$$r^2 - c_1 r - c_2 = 0.$$

Two cases exist:

a) $r_1 \neq r_2$. Then, all the solutions to Eq. (1.1) take the form

$$x_n = \alpha_1 r_1^n + \alpha_2 r_2^n,$$

where α_1 and α_2 are obtained from the initial conditions, by solving the equations

$$\begin{cases} \alpha_1 + \alpha_2 &= x_0 \\ r_1\alpha_1 + r_2\alpha_2 &= x_1. \end{cases}$$

b) $r_1 = r_2$. Then, all the solutions to Eq. (1.1) take the form

$$x_n = (\alpha + \beta n)\, r_1^n,$$

where α and β are obtained from the initial conditions, by solving the equations

$$\begin{cases} \alpha &= x_0 \\ r_1\alpha + r_2\beta &= x_1. \end{cases}$$

2. Exercises of Homework 7

Exercise 1. Solve Exercise 8 p. 423 of [1]. A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the two previous years.

a) Find a recurrence relation for $\{L_n\}$, where L_n is the number of lobsters caught in year n, under the assumption for this model.

b) Find L_n if $100,000$ lobsters are caught in year 1 and $300,000$ were caught in year 2.

Exercise 2. Somewhat like Exercise 4 p. 467 of [1]: Every day at noon, a population census of an aphid[1] colony is done. Every afternoon, each aphid bears two offsprings. Every morning, all aphids older than 24 hours die. On the first day at noon, the colony had just 30 young aphids that had been born that morning.

a) Compute the population of the aphid colony on the 2nd, 3rd, 4th and 5th day at noon.
b) Write a recurrence relation describing the number of aphids in the colony on day n.
c) Solve this recurrence relation to give a formula for the population on day n.
d) On what day will the population reach 100,000?

Exercise 3. Solve the recurrence relation $x_n = 2x_{n-2} - x_{n-4}$, when $x_0 = 2$, $x_1 = 1$, $x_2 = 2$, $x_3 = \frac{3}{2}$.

Hint: Solve the recurrence relation separately for n odd and for n even. As usual, compute the first few terms of the sequence, to see what it looks like.

References

[1] K. H. Rosen. *Discrete Mathematics and Its Applications*. Mc Graw Hill, 5 edition, 2003.

[1] An aphid is a small insect.