CS275 GRADED HOMEWORK 6 - PROVISIONAL SOLUTION

GIVE BACK ON THURSDAY NOVEMBER 4TH 2004 AT BEGINNING OF CLASS

For each question, read **each word** with the greatest care and **without hurrying**. If you have doubts about what is asked, **go back** to the wording of the question until the meaning of the question is clear. Then try to find an answer. If you get stuck, don't hesitate to **contact** your T.A. or me.

Please write your section number on your homework as well as a rough estimate of the time you spent solving it.

Exercise 1. Solve Exercise 6 p. 718 of [1]. Construct circuits from inverters, AND gates, and OR gates to produce these outputs.
Solution: See Figure 2.1

- **a)** $\bar{x} + y$.
- **b)** $\overline{(x + y)x}$ .... You may also note that this is 0.
- **c)** $\overline{xyz + \bar{x}\bar{y}\bar{z}}$.
- **d)** $\overline{(\bar{x} + z)(y + \bar{z})}$.

Exercise 2. Solve Exercise 8 p. 718 of [1]. Design a circuit for a light fixture controlled by four switches where flipping one of the switches turns the light on when it is off and turns it off when it is on.

Solution: Since $F(x, y) = xy + \bar{x}\bar{y}$ "flips" its output each time one of its inputs are flipped, one can hope that $F(F(w, x), F(y, z))$ behaves in the same way. It is easy to check with a truth table that this is indeed the case. So the light fixture circuit implements (Fig. 2.1) the boolean expression $(wx + \bar{w}\bar{x})(yz + \bar{y}\bar{z}) + \overline{(wx + \bar{w}\bar{x})}\;\overline{(yz + \bar{y}\bar{z})}$.

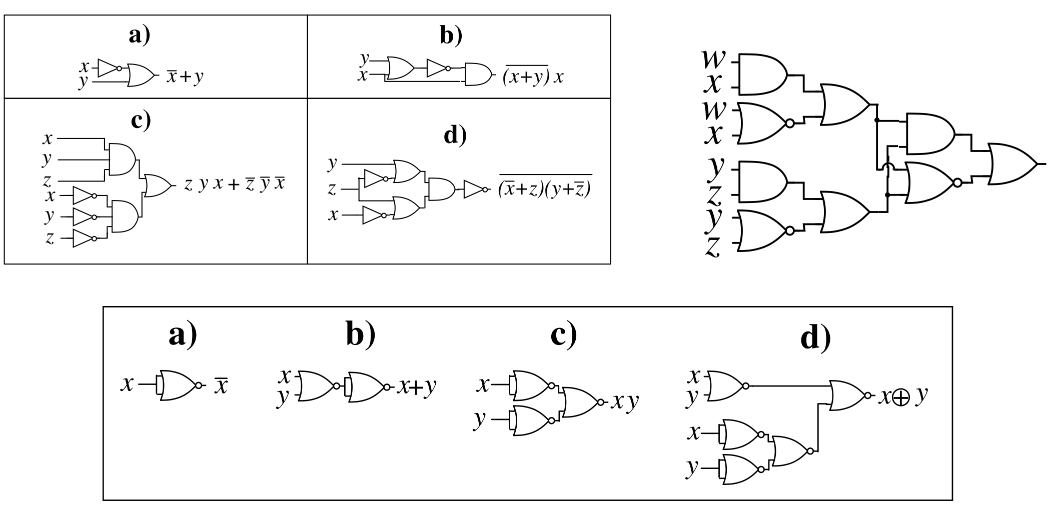

FIGURE 2.1. Top left: Solution to Ex. 1. Top right: Solution to Ex. 2. Bottom: Solution to Ex. 3.

Exercise 3. Solve Exercise 16 p. 718 of [1]. Use NOR gates to construct circuits with these outputs.

a) $\bar{x}$ .... $\bar{x} = x \downarrow x$.
b) $x + y$ .... $x + y = \overline{\overline{x + y}} = \overline{x \downarrow y} = (x \downarrow y) \downarrow (x \downarrow y)$.
c) xy .... $xy = \overline{\overline{xy}} = \overline{\bar{x} + \bar{y}} = \overline{(x \downarrow x) + (y \downarrow y)} = (x \downarrow x) \downarrow (y \downarrow y)$.
d) $x \oplus y$ .... $x \oplus y = \overline{xy + \bar{x}\bar{y}} = ((x \downarrow x) \downarrow (y \downarrow y)) \downarrow (x \downarrow y)$.

Exercise 4. Solve Exercise 4 p. 409 of [1]. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if

a) $a_n = 0$. .... $-3 \cdot 0 + 4 \cdot 0 = 0$.
b) $a_n = 1$. .... $-3 \cdot 1 + 4 \cdot 1 = 1$.
c) $a_n = (-4)^n$
$-3 \cdot (-4)^{n-1} + 4 \cdot (-4)^{n-2} = -3 \cdot (-4)^{n-1} - (-4) \cdot (-4)^{n-2} = (-3 - 1) \cdot (-4)^{n-1} = (-4)^n$.
d) $a_n = 2\,(-4)^n + 3$.
$-3 \cdot \left(2\,(-4)^{n-1} + 3\right) + 4 \cdot \left(2\,(-4)^{n-2} + 3\right) = -2 \cdot 3 \cdot (-4)^{n-1} - 2 \cdot (-4) \cdot (-4)^{n-2} + 3 = 2\,(-4)^n + 3$.

Exercise 5. Solve Exercise 12 p. 409 of [1]. Assume that the population of the world in 2002 is 6.2 billion and is growing at the rate of 1.3% a year.

a) Set up the recurrence relation for the population of the world n years after 2002.
Solution: Let x_n be the population of the world n years after 2002, so that $x_0 = 6.2e9$. Since the growth rate is 1.3%, one has $x_{n+1} = 1.013 \cdot x_n$.

b) Find an explicit formula for the population of the world n years after 2002.
Solution: $x_n = x_0 \cdot 1.013^n$.

c) What will the population of the world be in 2022?[1]
Solution: $x_{21} = x_0 \cdot 1.013^{20} \simeq 6.2e9 \cdot 1.295 \simeq 8.028e9$.

Exercise 6. Solve Exercise 28 p. 410 of [1].

a) Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one, two or three stairs at a time.
Solution:
Let´s call x_n the number of ways a person can climb n stairs.
- $x_1 = 1$ since a single stair can only be climbed in 1 step.
- $x_2 = 2$ since 2 stairs can be climbed in (a) 2 single steps or (b) a double step.
- $x_3 = 4$ since three stairs can be climbed in (a) three single steps, (b) a single and a double step (c) a double and a single step, or (d) a triple step.
- In general, I can climb n steps by (a) climbing $n-1$ steps and adding a single step, (b) climbing $n-2$ steps and adding a double step, or (c) climbing $n-3$ steps and adding a triple set. These cases are disjoint and cover all possible cases. Thus one has $x_n = x_{n-1} + x_{n-2} + x_{n-3}$.

b) What are the initial conditions?
Solution: See above.

c) How many ways can this person climb a flight of eight stairs?
Solution: The sequence is thus $x_4 = 7$, $x_5 = 13$, $x_6 = 24$, $x_7 = 44$, $x_8 = 81$.

References

[1] K. H. Rosen. *Discrete Mathematics and Its Applications.* Mc Graw Hill, 5 edition, 2003.

[1] Assuming the author´s model is correct.