CS275 GRADED HOMEWORK 3

GIVE BACK ON TUESDAY SEP. 28TH 2004 AT BEGINNING OF CLASS

For each question, read **each word** with the **greatest care** and **without precipitation**. If you have doubts about what is asked, **go back** to the wording of the question until the meaning of the question is clear. Then try to find an answer. If you get stuck, don't hesitate to **contact** your T.A. or me.

Please write on your homework your section number and an estimate of the time you spent solving it.

Exercise 1.
Consider the rectangular city maps in Figure 1.1.

- **a)** How many distinct ways are there to go from point A to point B, if one is only allowed to move along the lines, either Southward or Eastward.
- **b)** How many distinct ways are there to go from point C to point D, if one is only allowed to move along the lines, either Southward or Eastward.

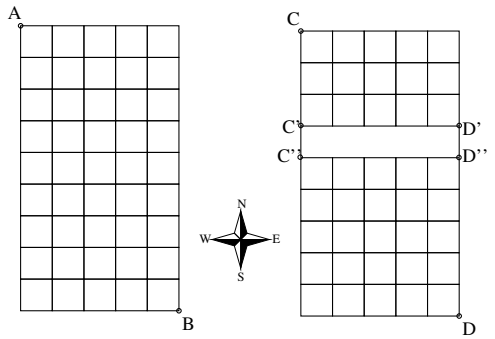

FIGURE 1.1. **Left:** Rectangular city plan. **Right:** Two-part city with two bridges.

Exercise 2. A lottery has 64 tickets. How many ways are there of giving 60 of these tickets to 60 persons, knowing that each person receives exactly one ticket? Justify your answer.

Exercise 3. A web server answers 1000 queries per day. For each, a single line of text is added to a log file. At 1am, the server is put offline for a minute. During this time, half[1] the lines of the log file are deleted. Let x_m be the number of lines in the log file at 1:01am of the m^{th} day of operation of the server.

[1]Assume it is possible to keep a fractional number of lines.

a) Taking into account the 1000 daily queries and the deletion process, write x_{m+1} as a function of x_m.
b) On the first day, at 1:01am, the log file has 1000 lines, i.e. $x_1 = 1000$.
1) Compute x_2, x_3 and x_4.
2) What are x_{1000}, $x_{1000000}$ and $x_{1000000000}$?
3) Justify your answer to the last question with a proof by induction.

Exercise 4. For all $n \in \mathbb{N}$, define $x_n = 1 + \frac{1}{2} + \ldots + \frac{1}{2^n} = \sum_{i=0}^{n} \frac{1}{2^i}$.

a) Compute x_0, x_1, x_2, x_3 and x_4.

b) Show by induction that, for all $n \in \mathbb{N}$, $x_n = 2 - \frac{1}{2^n}$.

Exercise 5. The passwords on a computer system consist of characters. Each character belongs to the set $\mathcal{C} = \{$`'a'`...`'z'`, `'A'`...`'Z'`, `'0'`...`'9'`$\}$.

a) Write in mathematical notation the set of 4-character passwords. How many elements does it have?
b) Write in mathematical notation the set of 4-, 5- or 6- character passwords? What is its cardinal?
c) Write in mathematical notation the set of 4-character passwords consisting only of upper- or lower-case letters? How many such passwords are there?
d) Write in mathematical notation the set of 4-character passwords with 1 or more digits in them? How many such passwords are there?
e) Write in mathematical notation the set of 4-character passwords with exactly 3 digits in them? How many such passwords are there?

Exercise 6. Solve Exercise 8 p. 310 of the textbook [1]: how many different three-letter initials with none of the letters repeated can people have?

Exercise 7. Solve Exercise 26 p. 311 of the textbook [1]: how many license plates can be made using either three letters followed by three digits or four letters followed by two digits?

Exercise 8. Solve Exercise 20 p. 325 of the textbook [1]: how many bit strings of length ten have

a) exactly three 0s?
b) more 0s than 1s?
c) at least seven 1s?
d) at least three 1s?

Exercise 9. How many sequences $(a_1, a_2, a_3, a_4, a_5)$ of five numbers taken in $\{0, ..., 9\}$ are such that $a_1 < a_2 < a_3 < a_4 < a_5$? Justify your answer.

References

[1] K. H. Rosen. *Discrete Mathematics and Its Applications.* Mc Graw Hill, 5 edition, 2003.